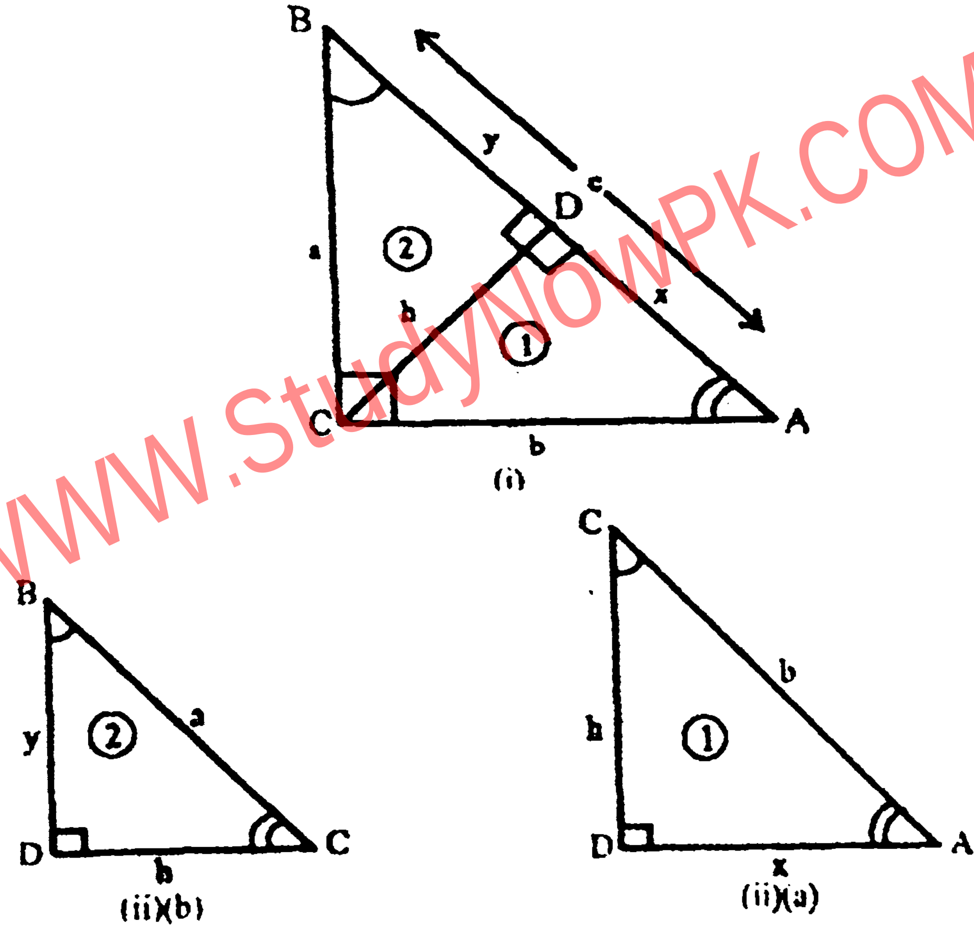
Unit 15
Pythagoras Theorem

THEOREM 15.1.1
PYTHAGORAS THEOREM

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



Given:

ΔACB is a right angle triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To Prove:

$c^2 = a^2 + b^2$

Construction:

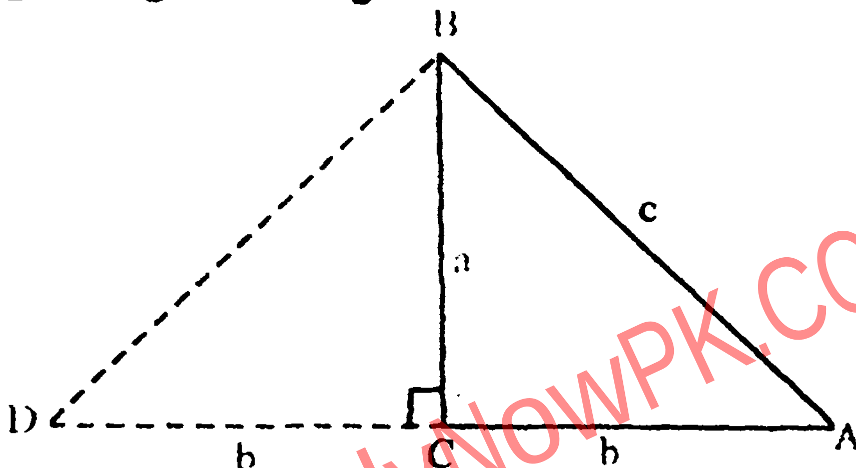
Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$. Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment $\overline{CD}$ splits ΔABC into two triangles ADC and BDC which are separately shown in figure ii (a) and ii (b) respectively.

Proof:

Statements	Reasons
In the correspondence $\Delta ADC \leftrightarrow \Delta ACB$	
$\angle A \cong \angle A$	Refer to figure ii (a) and (i) common-self congruent
$\angle ADC \cong \angle ACB$	Construction given both measure 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$
$\therefore\ \Delta ADC \cong \Delta ACB$	Congruency of three angles
$\therefore\ \frac{x}{b} = \frac{b}{c}$	Measure of corresponding sides of similar triangles is similar.
Again in the correspondence $\Delta BDC \leftrightarrow \Delta BCA$	Refer to figure ii(b) and (i)
$\angle B \cong \angle B$	Common self congruent
$\angle BDC \cong \angle BCA$	Construction given, both measure 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$, complements of $\angle B$
$\therefore\ \Delta BDC \cong \Delta BCA$	Congruency of three angles.
$\therefore\ \frac{y}{a} = \frac{a}{c}$	Sides of similar triangles are proportional. (Theorem 6)
or $y = \frac{a}{c^2}$ ..... (ii)	
But $y + x = c$	Supposition
$\frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both sides with c.
or $c^2 = a^2 + b^2$	

THEOREM 15.1.2
Converse of PYTHAGORAS THEOREM 15.1.1

In a triangle if the sum of the squares of the I measures of two sides is equal to the square of the measure of the third side, the triangle is a right angled triangle.



Given:

In a ΔABC, $m\overline{AB} = c$, $m\overline{BC} = a$ and $m\overline{AC} = b$ such that $a^2 + b^2 = c^2$

To Prove:

$m\angle ACB = 90°$, ΔACB is a right angled triangle.

Construction:

Draw $\overline{CD}$ Perpendicular to $\overline{BC}$ such that $\overline{CD} = \overline{CA}$. Join B and D.

Proof:

Statements	Reasons
ΔDCB is a right angled triangle	Construction
$(m\overline{BD})^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore \quad (m\,\overline{BD})^2 = c^2$	
or $m\,\overline{BD} = c$	Taking square root of both sides.

$\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each is equal to c
$\therefore \quad \Delta DCB \cong \Delta ACB$	$SSS \cong SSS$
$\therefore \quad \angle DCB \cong \angle ACB$	Corresponding angles of congruent triangles.
But $m\angle DCB = 90°$	Construction
$\therefore \quad m\angle ACB = 90°$ and the Δ ACB is a right angled triangle.	

EXERCISE 15.1

Q1. Verify that the Δs having the following measures of sides are right – angled.

(i) $a = 5cm,\ \ b = 12cm,\ \ c = 13$

Solution:

By Pythagoras theorem

$$a^2 + b^2 = (5)^2 + (12)^2$$
$$= 25 + 144 = 169$$
$$c^2 = (13)^2 = 169$$
$$\therefore \quad a^2 + b^2 = c^2$$

Thus the triangle is right angled triangle.

(ii) $a = 1.5\ cm, b = 2cm, 2.5cm$

Solution:

By Pythagoras theorem

$$a^2 + b^2 = (1.5) + (2)$$
$$= 2.25 + 4 = 6.25$$
$$c^2 = (2.5)^2 = 6.25$$
$$a^2 + b^2 = c^2$$

Thus the triangle is right angled triangle.

(iii) $a = 9cm, b = 12cm, c = 15cm$

Solution:

By Pythagoras theorem

$$a^2 + b^2 = (9)^2 + (12)^2$$
$$= 81 + 144 = 225$$

$$c^2 = (15) = 225$$
$$\therefore \quad a^2 + b^2 = c^2$$

Hence the triangle is right angled triangle.

(iv) $\boldsymbol{a = 16cm, b = 30cm, c = 34cm}$

Solution:

By Pythagoras theorem

$$a^2 + b^2 = (16)^2 + (2)^2$$
$$= 256 + 900 = 1156$$
$$c^2 = (34)^2 = 1156$$
$$\therefore \quad a^2 + b^2 = c^2$$

Hence the triangle is right angled triangle.

Q2. Verify that $\boldsymbol{a^2 + b^2}$, $\boldsymbol{a^2 - b^2}$ and $\boldsymbol{2ab}$ are the measures of the sides of a right angled triangle where $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two real numbers $(a > b)$

Solution:

Let ABC be triangle such that

$$\overline{AB} = a^2 + b^2$$
$$\overline{BC} = a^2 - b^2$$
$$\overline{AC} = 2ab$$

By Pythagoras theorem

$$|\overline{AB}|^2 = (a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2$$

and $$|\overline{AC}|^2 + |\overline{BC}|^2 = (2ab)^2 + (a^2 - b^2)^2$$
$$= 4a^4 + a^4 + b^4 - 2a^2b^2$$
$$= a^4 + b^4 + 2a^2b^2$$

So $$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$

Hence ABC is a right angled triangle.

Q3. The three sides of a triangle are of measure $\boldsymbol{8, x}$ and $\boldsymbol{17}$ respectively. For what value of $\boldsymbol{x}$ will it become base of a right angled triangle?

Solution:

If x is the base of right angled triangle then -17 is the measure of hypotenuse.

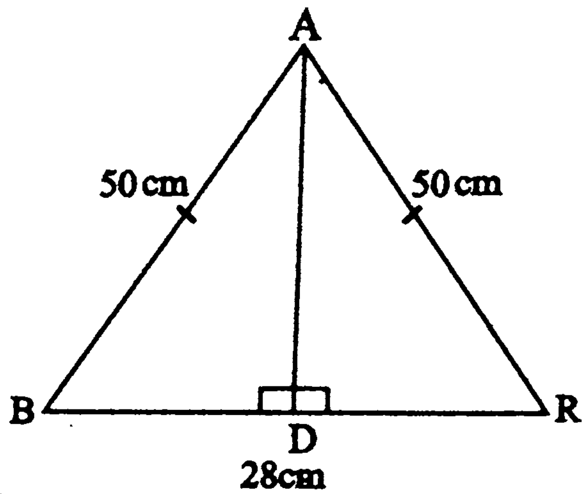
By Pythagoras Theorem

$$(hypotenus)^2 = (base)^2 + (perpendicular)^2$$
$$(17)^2 = (x)^2 + (8)^2$$
$$289 = x^2 + 64$$
$$x^2 = 289 - 64 = 225$$
$$x = \sqrt{225} = 15$$

Q4. In an isosceles Δ, the base $\overline{BC} = 28\ cm$, and $\overline{AB} = \overline{AC} = 50\ cm$. If $\overline{AD} \perp \overline{BC}$, then find

(i) length of $\overline{AD}$ (ii) Area of ΔABC

Solution:



(i) $\overline{AD} \perp \overline{BC}$

$\therefore$ D is mid point for $\overline{BC}$

So $m\overline{BD} = \frac{1}{2}(28) = 14\ cm$

From right angled ΔABD

$(m\overline{AB})^2 = (m\overline{BD})^2 + (m\overline{AD})^2$

$(50)^2 = (14)^2 + (m\overline{AD})^2$

$(m\overline{AD})^2 = (50)^2 - (14)^2 = 2500 - 196 = 2304$

$m\overline{AD} = \sqrt{2304} = 48cm$

(ii) **Area of ΔABC**

$= (m\overline{BC})\,(m\overline{AD})$

$= (28)(48) \quad = 672cm^2$

Q5. In a quadrilateral ABCD, the diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other. Prove that

$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$$

Solution:

The diagram AC and BD of the quadrilateral ABCD meet at O perpendicularly in the right triangles ΔAOB

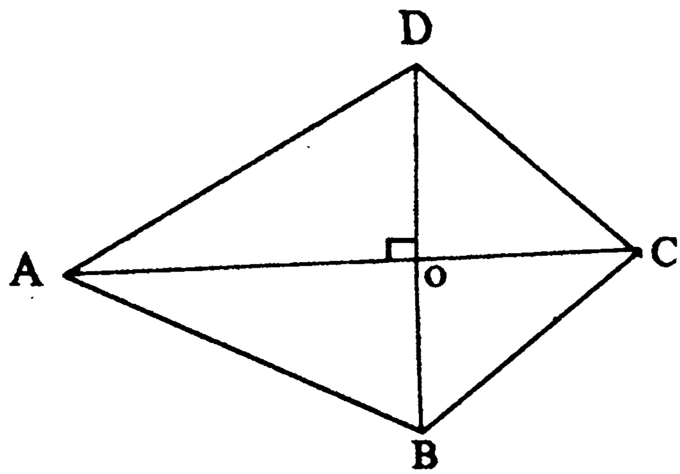
$$m\overline{AB}^2 = m\overline{AO}^2 + m\overline{OB}^2 \qquad \text{(i)}$$

In the right triangle ΔBOC

$$m\overline{BC}^2 = m\overline{OB}^2 + m\overline{OC}^2 \qquad \text{(ii)}$$

In the right triangle DOC

$$m\overline{DC}^2 = m\overline{OC}^2 + m\overline{OD}^2 \qquad \text{(iii)}$$



Adding (i) and (ii)

$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{OA}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$

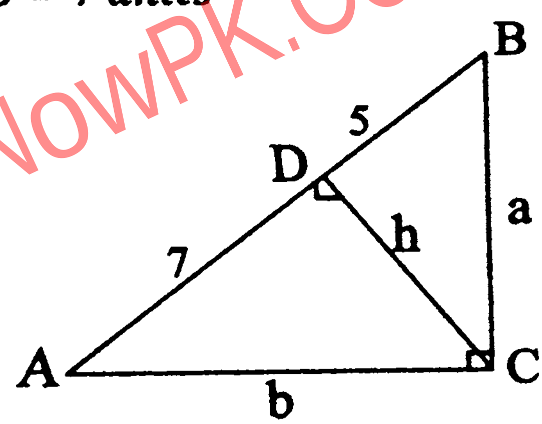
Adding (i) and (iv)

$m\overline{AD}^2 + m\overline{BC}^2 = m\overline{OA}^2 + m\overline{OB}^2 + m\overline{OC}^2 + m\overline{OD}^2$

Hence $m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$

Q6. (i) In the ΔABC as shown in the figure, $m\angle ACB = 90^0$ and $\overline{CD} \perp \overline{AB}$. Find the lengths a, h and b if $m\overline{BD} = 5\ units$ and $m\overline{AD} = 7\ units$

Solution:



$m\overline{AB} = 5 + 7 = 12$

In right angled ΔBDC

$a^2 = 25 + h^2$ .... (1)

In right angled ΔADC

$b^2 = 49 + h^2$ .... (2)

In right angled ΔABC

$a^2 + b^2 = 144$ ..... (3)

Adding (1) and (2)

$a^2 + b^2 = 74 + 2h^2$...... (4)

From (3) and (4)

$74 + 2h^2 = 144$

$2h^2 = 144 - 74 = 70$

$h^2 = 35$

$h = \sqrt{35}\ units$

Put $h^2 = 35$ in (1)

$a^2 = 25 + 35 = 60$

$a = \sqrt{60} = 2\sqrt{15}\ units$

Put $h^2 = 35$ in (2)

$b^2 = 49 + 35$

$b^2 = 84$

$b = \sqrt{84} = 2\sqrt{21}\ units$

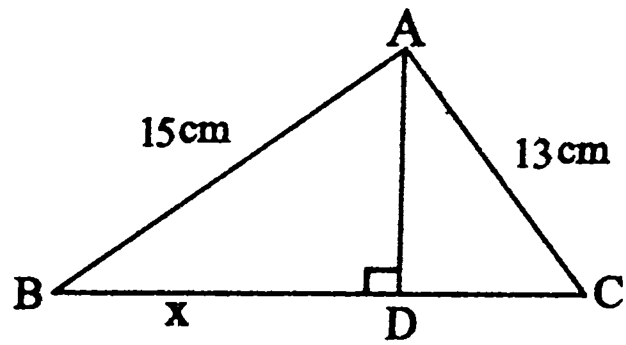
So $a = 2\sqrt{15}\ units$

$h = \sqrt{35}\ units$

$b = 2\sqrt{21}\ units$

(ii) Find the value of x in the shown figure.

Solution:



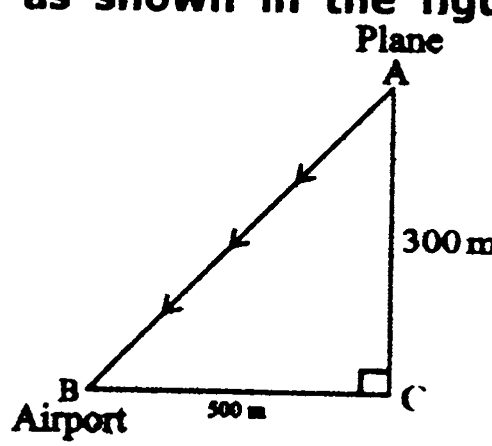
From ΔADC

$$(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{DC})^2$$
$$(13)^2 = (m\overline{AD})^2 + (5)^2$$
$$169 = (m\overline{AD})^2 + 25$$
$$(m\overline{AD})^2 = 169 - 25 = 144$$
$$\therefore \quad m\overline{AD} = 12cm$$

From ΔABD

$$(m\overline{AB})^2 = (m\overline{AD})^2 + (m\overline{BD})^2$$
$$(15)^2 = (12)^2 + (x)^2$$
$$225 = 144 + x^2$$
$$x^2 = 225 - 144 = 81$$
$$\therefore \quad x = 9\ cm$$

Q7. A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport?



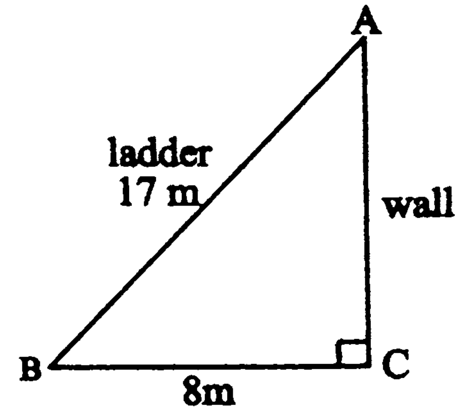
Solution:

$m\overline{BC} = 500\ m$; $m\overline{AC} = 300\ m$

By Pythagoras theorem

$$m\overline{AB}^2 = m\overline{BC}^2 + m\overline{AC}^2$$
$$m\overline{AB}^2 = (500)^2 + (300)^2$$
$$= 250000 + 90000$$
$$= 340000 \qquad = \sqrt{34 \times 10000}$$
$$m\overline{AB} = 100\sqrt{34}\ m$$

Q8. A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach?



Solution:

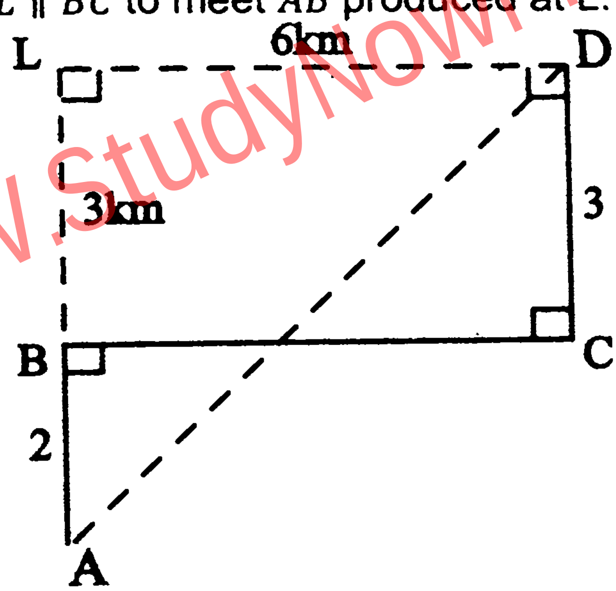
By Pythagoras Theorem

$$(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$$
$$(17)^2 = (m\overline{AC})^2 + (8)^2$$
$$(m\overline{AC})^2 = (17)^2 - (8)^2$$
$$= 289 - 64 = 225$$
$$m\overline{AC} = \sqrt{225} = \mathbf{15\ cm}$$

Q9. A student travels to his school by the route as shown in the figure. Find $m\ \overline{AD}$, the direct distance from his house to school.

Solution:

A is house, B is bus stop and D is school. Produce $\overline{AB}$ and draw $\overline{DL} \parallel \overline{BC}$ to meet AB produced at L.



We have to find AD.

$$m\overline{LD} = m\overline{BC} = 6\ km$$
$$m\overline{BL} = m\overline{CD} = 3km$$
$$m\overline{AL} = m\overline{AB} + m\overline{BL} = 2 + 3 = 5\ km$$

ΔALD is a right angled Δ

By Pythagoras theorem

$$m\overline{AD}^2 = m\overline{AL}^2 + m\overline{LD}^2$$
$$= (5)^2 + (6)^2 = 25 + 36 = 61$$
$$m\overline{AD} = \sqrt{\mathbf{61}}\ \mathbf{km}$$

REVIEW EXERCISE 15

Q1. Which of the following is true and which are false?

(i) In a right angled triangle greater angle is of 90^0.

(ii) In a right angled triangle right angle is of 60^0.

(iii) In a right triangle hypotenuse is a side opposite to right angle.

(iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm.

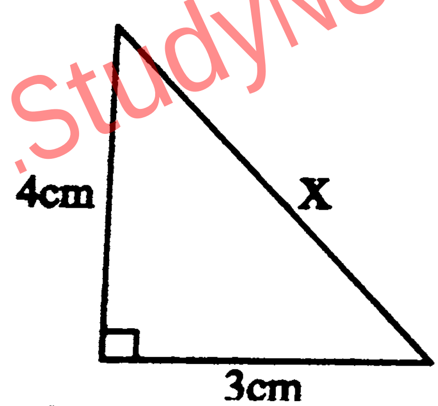
(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.

Answers:

(i) T	(ii) F	(iii) T	(iv) T	(v) T	(vi) F

Q2. Find the unknown value in each of the following figures.

(i)

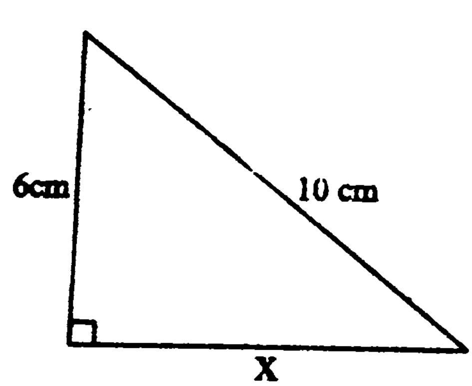


By Pythagoras Theorem

$$x^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$x^2 = \sqrt{25} = 5\ cm$$

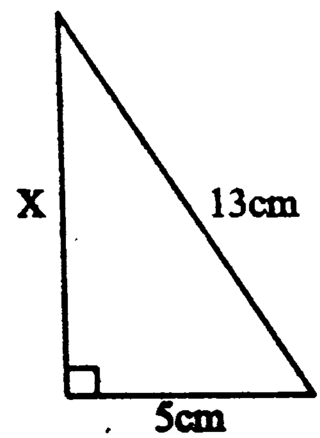
(ii)



By Pythagoras Theorem

$$(10)^2 = (6)^2 + (x)^2$$
$$100 = 36 + x^2$$
$$x^2 = 100 - 36 = 64$$
$$x^2 = \sqrt{64} = 8\ cm$$

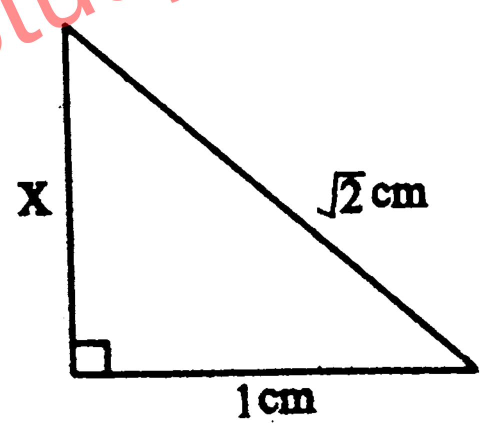
(iii)



By Pythagoras Theorem

$$(13)^2 = (x)^2 + (5)^2$$
$$169 = x^2 + 25$$
$$x^2 = 169 - 25$$
$$x = \sqrt{144} = 12\ cm$$

(iv)



By Pythagoras Theorem

$$(\sqrt{2})^2 = (x)^2 + (1)^2$$
$$2 = x^2 + 1$$
$$x^2 = \sqrt{1} = 1\ cm$$